SA-MR-9004

STATISTICAL DECISIONS UTILIZING NEURAL NETS

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MR. GEORGE SCHLENKER

30 MAY 1990

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This report documents the development of neural nets that determine if the probability distribution for sample data is unimodal or multimodal and if there is one stochastic component or more than one stochastic component.

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SUBJECT: Statistical Decisions Utilizing Neural Nets.

1. Summary:

Neural networks were developed that accurately determine the statistical characteristics: modality and number of stochastic components of underlying probability distribution(s) for sample data. Sample data examples, used to teach the neural nets were generated utilizing either a single beta distribution or a mixture of beta distributions. Once the neural net learned to distinguish between unimodal and multimodal examples and also between unimodal and mixture densities, they were challenged with unknown test cases. The test cases were also generated from either a single beta distribution or a mixture of beta distributions. Therefore the initial test results apply to a restricted class of distributions having bounded domains. However these trained networks were furnished to Mr. Schlenker who challenged the networks with sample data from distributions other than beta, thus widening the application domain of the networks. An explanation of this additional work is detailed in appendix A.

The initial testing of the neural networks consisted of 40 unknown sample data examples generated utilizing beta distributions. The results of these tests are: (1) correctly identified 39 out of the 40 as being either unimodal or multimodal, an accuracy of 97.5 percent. This exceeds the accuracy of currently available statistical methods; (2) correctly identified 36 out of 40 as having either one component or more than one component, an accuracy of 90 percent. There are no statistical methods available for determination of components. For the larger class of distributions, the corresponding accuracy rates are 93 percent and 81 percent.

Mr. Manata's work in developing the neural networks was originally published in memorandum report SA-MR-9002. Mr. Schlenker's work on the extensions was originally published in memorandum report SA-MR-9003. But, since these two reports are interrelated, they are being consolidated in this report.

2. Neural Network Discussion:

a. Knowledge:

There are at least three ways of representing knowledge in a computer environment: standard computer programs, expert systems, and neural networks.

A standard program has two types of knowledge: instructions, and the value of the variables used by the program If a user wants to know what knowledge the computer contains, the list of instructions and the current value of the variables provides this information.

An expert system has three types of knowledge: (1) if - then rules, (2) initial facts and beliefs, (3) conclusions generated by the if - then rules As in the standard program a user can determine the systems knowledge by listing the current rule, facts, beliefs, and the value of any variables that the system uses.

Neural networks contain two types of knowledge: the network, nodes and the connections between nodes, and the weights associated with each connection. If the user has a question concerning the computer's knowledge, the network and the weights are available. But, the user would not find it easy to ascertain what this knowledge has to do with solving the problem. As an example of neural network knowledge, figure 1.0 shows the type of network that was used to solve the statistical decision problem, and table 1.0 is a partial listing of the weight values.

FIGURE 1.0
NETWORK ARCHITECTURE

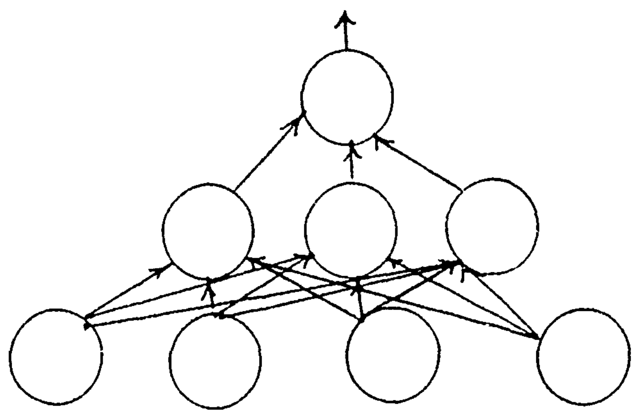

TABLE 1.0
CONNECTION WEIGHTS

INPUT LAYER NODES	MIDDLE LAYER NODES 1	2	3
1	-.175	-.890	-.283
2	-.479	+.415	-2.177
3	-1.117	-1.003	+6.535
4	-1.123	+.282	-7.678

MIDDLE LAYER NODES	OUTPUT LAYER NODE 1
1	+.886
2	-.849
3	+19.521

b. Neural Networks:

Neural networks, of the type shown in figure 1.0, have three or more layers of nodes with the nodes of one layer connected to all the nodes of the next layer. Neural network problem solving is initiated by providing the first layer, the input layer, with a vector of values, the input vector, containing information which describes the problem to be solved. Each input node receives one component of this vector and feeds it into the connections between itself and every middle layer node. The connections multiply these components, by the connection weights, and deliver the modified component values to the middle layer nodes. Each middle layer node sums its incoming values and operates on the sum with an activation function, usually a sigmoid function. This generates a nodal output value which is furnished to the connections between the middle layer nodes and every output node. The multiplication process that occurred between the input layer and middle layer is repeated between the middle and output layers. The output nodes sum the incoming values, operate on the sum with the activation function, and generate output values. These output values are the answer to the the problem.

Before a network can solve a specific problem it has to be taught how to solve the set of problems of which the specific problem is a member. This requires that the network be furnished representative examples of the problem set, and the answer for each example. During training, the network compares its answer with the correct answer. If its answer is within a specified range of the correct answer, the network is considered to have "learned" to solve the set of problems. This, delta value, set prior to training, is the maximum error, between the correct and network answers, that the developer will accept. Delta is usually set at 0.1 but it can be any value greater than zero and less than one. Until this criterion is met, the network back propagates the actual error through the network to modify the connection weights. This feed-forward-back-propagation process continues until the network has achieved "learning" as defined by convergence to a given limit. Once the network has "learned," it is tested to determine its accuracy.

c. Neural Net Simulation:

Neural networks are a parallel processing technique. But, they can be emulated on sequential computers. This is accomplished by utilizing simulation software. AMSMC-SAO has simulation software (NETS) developed by NASA for IBM-compatible PCs.

d. Statistical Decision Making:

Statistical aids utilizing neural networks were suggested to Mr. J. Manata of AMSMC-SAO by Mr. G. Schlenker of AMSMC-SAS. The statistical questions concerned the modality, and the number of stochastic components (the number of unimodal components in a probability mixture model) of probability distributions producing sample data. Statistical methods exist for determining modality but not for number of components.

3. Methodology:

a. Net Architecture:

The sample data was distributed over 20 histogram cells anticipating that the neural networks would be used with samples of 200 or more data points; therefore, 20 cells seemed a reasonable number to assure an adequate population for each cell. The number of output nodes was set at either one or two depending on the type of problem the network was required to solve. If the network was required to solve a modality or component problem, the number of outputs was one. If the network was required to solve two problems, modality and components, the number of outputs was two. The number of middle layer nodes and the connections between the input layer, the middle-layer, and the output layer was determined by trial and error. Nineteen net architectures were tested. Table 2.0 lists the different architectures, and figure 2.0 shows examples of the architectures.

The architecture that worked the best had twenty input nodes, seven middle nodes, and one or two output nodes. An example of this architecture is shown in figure 2b.

NUMBER OF INPUT NODES	NUMBER OF MIDDLE LAYERS	TOTAL NUMBER OF MIDDLE LAYER NODES	NUMBER OF MIDDLE LAYER NODES PER MIDDLE LAYER	NUMBER OF OUTPUT NODES	FIGURE
20	0	0	0	2	8a
20	1	3	3	2	8b
20	1	4	4	2	8b
20	1	5	5	2	8b
20	1	7	7	2	8b
20	1	9	9	2	8b
20	1	10	10	2	8b
20	1	11	11	2	8b
20	1	12	12	2	8b
20	2	7	4,3	2	8c
20	2	7	3,4	2	8c
20	2	7	4,3	2	8d
20	2	7	3,4	2	8d
20	2	7	4,3	2	8e
20	2	7	5,2	2	8c
20	2	7	2,5	2	8c
20	2	7	5,2	2	8e

TABLE 2.0
LISTING OF NETWORK ARCHITECTURES ATTEMPTED

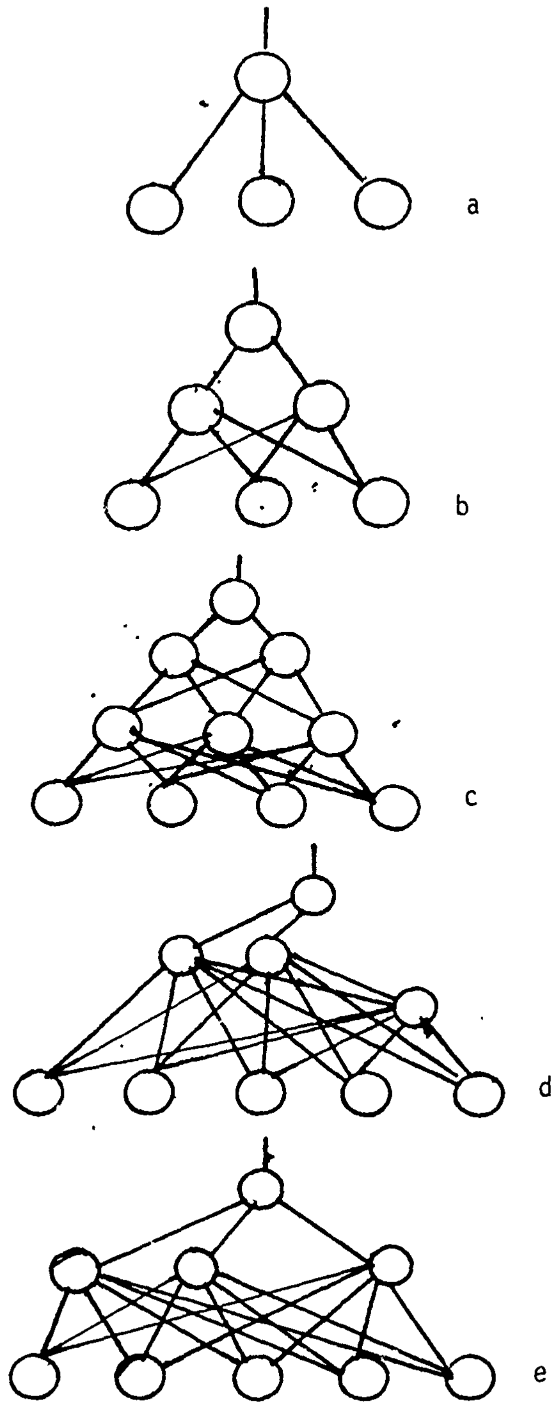

FIGURE 2.0
EXAMPLES OF NETWORK ARCHITECTURE

b. Generation of Training and Test Data Sets:

The training sample data distributions, generated by Mr. Schlenker, were developed by a Monte-Carlo selection of random variables from beta distributions or mixtures of beta distributions [a]. Beta distributions were used because they provide the capability for the generation of a variety of density function shapes, and they have a well-defined finite domain.

Each sample consisted of 400 data points which were distributed over 20 histogram cells. Two separate training sets were generated; one set had examples containing sampling error, noise, and the other, an idealized set, did not. The idealized case represents an essentially infinite sample. The use of two training sets was to determine if the network was more accurate when trained with noisy or idealized data.

Mr. Schlenker also generated test examples utilizing one or more beta distributions, plus one example from a truncated weibull distribution. The number of data points in the test examples varied between 150 and 3200. Table 3.0 lists the training sets and table 4.0 lists the test sets. Table 5.0 shows the location of the modes and provides a relative measure of their separation in terms of the standard deviation. Additionally, the coefficient of skewness (beta) of the distribution is shown to suggest the range of this parameter that the network had to recognize. With the exception of seven sets, all the sets were generated using a Simscript random number seed index 1.

Table 6.0 lists the parameter values of the beta mixture which produce each set. For the unimodal, one-component sets, the threshold parameter is always 0, and the upper limit is 1; i.e., the distribution form is standardized beta. In the case of two stochastic components, either one or two modes are produced, depending on the parameters of each stochastic component and on the mixture parameter (r). In all cases, save two, the threshold and limit parameters are 0 and 0.7, for component 1, and 0.3 and 1.0 for component 2. The two exceptions are sets HM36 and HM41. For HM36 these parameters are (0, 0.5), for component 1, and (0.3, 1.0) for component 2. For HM41 these parameters are (0, 0.6), for component 1, and (0.4, 1.0) for component 2. As is seen, the lower and upper limits on the domain of all mixtures are 0 and 1. These are also the respective histogram limits.

---

[a] The density function for a mixture of beta densities is given for the random variable x as

$$f(x) = r w^{A1-1} (1-w)^{B1-1} / CB(A1,B1) + (1-r) z^{A2-1} (1-z)^{B2-1} / CB(A2,B2) ,$$

where r is proportion of first component and where CB(a,b) is the complete beta function with parameters a and b, and where A1, B1 are parameters of the first component and A2, B2 are parameters of the second. Auxiliary variables w, z are given in terms of x as $w = (x - Th1)/(Ul1 - Th1)$ and $z = (x - Th2)/(Ul2 - Th2)$, for w and z limited to the unit interval. Threshold and upper limit parameters for component c are Thc and Ulc.

c. Scaling of Nodal Input Values:

A requirement of the NETS is that the nodal input values lie in the range of (.1,.9). Because of this requirement, the histogram of the training and test examples had to be modified to lie within this range. This scaling was accomplished by:

$$e = n/N + 0.1$$

where

e = scaled value
N = sample size
n = interval frequency

TABLE 3.0
TRAINING SETS

DISTRIBUTION USED TO GENERATE TRAINING SAMPLE	NUMBER OF DATA POINTS IN EACH SAMPLE	SAMPLE DISTRIBUTION	STOCHASTIC COMPONENTS	NUMBER OF TRAINING SETS
ONE BETA	400	UNIMODAL	1	30
TWO BETAS	400	UNIMODAL	2	9
TWO BETAS	400	MULTIMODAL	2	23

TABLE 4.0
TEST SETS

DISTRIBUTION USED TO GENERATE TEST SAMPLE	NUMBER OF DATA POINTS IS EACH SAMPLE	SAMPLE DISTRIBUTION	STOCHASTIC COMPONENTS	NUMBER OF TEST SETS
ONE BETA	150	UNIMODAL	1	4
ONE BETA	200	UNIMODAL	1	2
ONE BETA	400	UNIMODAL	1	13
ONE WEIBULL	3200	UNIMODAL	1	1
TWO BETAS	400	UNIMODAL	2	9
TWO BETAS	3200	UNIMODAL	2	2
TWO BETAS	400	MULTIMODAL	2	9

TABLE 5.0 Training Sets for Networks to Classify Samples of Density Functions with Respect to Modality and Number of Components

Set Name	Modes	Components	Loc Modes	Std Dev	Skewness	Seed
HU01	unimodal	1	0.00, ----	0.200	1.200	1
03			0.34, ----	0.200	0.286	
05			0.66, ----	0.200	-0.286	
09			0.00, ----	0.194	0.861	
13			0.34, ----	0.200	0.286	3
15			0.66, ----	0.200	-0.286	
19			0.00, ----	0.194	0.861	
22			0.25, ----	0.140	0.488	
24			0.50, ----	0.140	0.000	
35			0.50, ----	0.121	0.000	
37			0.70, ----	0.131	-0.364	
HM06	bimodal	2	0.35, 0.65	0.179	0.000	1
11			0.35, 0.65	0.176	0.237	
16			0.35, 0.65	0.176	-0.237	
26			0.35, 0.65	0.168	-0.476	
32			0.30, 0.65	0.197	0.486	
34			0.30, 0.65	0.198	0.247	
36			0.25, 0.65	0.222	0.242	
38			0.37, 0.65	0.184	0.150	
41			0.30, 0.70	0.214	0.000	
44			0.35, 0.65	0.172	0.252	
HM10	unimodal	2	0.50, ----	0.217	0.000	
14			0.45, ----	0.198	0.168	
15			0.48, ----	0.215	0.131	
18			0.62, ----	0.188	-0.196	
20			0.53, ----	0.215	-0.131	
22			0.35, ----	0.173	0.436	
25			0.44, ----	0.208	0.251	
27			0.65, ----	0.173	-0.436	
30			0.56, ----	0.208	-0.251	

TABLE 6.0 Parameters Which Characterize the Probability Distributions Used in Producing Training Sets

Set Name	Mix Param	Component 1 A	Component 1 B	Component 2 A	Component 2 B
HU01	---	0.6	2.4		
03	---	2.0	3.0		
05	---	3.0	2.0		
09	---	1.0	3.0		
13	---	2.0	3.0		
15	---	3.0	2.0		
19	---	1.0	3.0		
22	---	2.914	6.8		
24	---	5.878	5.878		
35	---	8.0	8.0		
37	---	8.0	4.0		
HM06	0.5	6.0	6.0	6.0	6.0
11	0.6	6.0	6.0	6.0	6.0
16	0.4	6.0	6.0	6.0	6.0
26	0.3	6.0	6.0	6.0	6.0
32	0.6	6.0	6.0	4.0	4.0
34	0.6	4.0	4.0	6.0	6.0
36	0.5	6.0	6.0	4.0	4.0
38	0.5	6.0	6.0	4.0	4.0
41	0.5	7.0	7.0	7.0	7.0
44	0.6	7.0	7.0	7.0	7.0
HM10	0.5	2.0	2.0	2.0	2.0
14	0.6	3.0	3.0	3.0	3.0
15	0.6	2.0	2.0	2.0	2.0
18	0.4	4.0	4.0	4.0	4.0
20	0.4	2.0	2.0	2.0	2.0
22	0.7	5.0	5.0	5.0	5.0
25	0.7	2.0	2.0	2.0	2.0
27	0.3	5.0	5.0	5.0	5.0
30	0.3	2.0	2.0	2.0	2.0

d. Training Sets:

The next step in the process of developing an accurate neural net, is the determination of the best set of training examples. The best set of training examples is that combination of examples, which results in a trained net, that provides the highest accuracy when the net is challenged with unknown test cases.

Examples of the training sets are shown in figures 3.0 - 8.0, figure 3.0 is the histogram for a unimodal distribution with one stochastic component; figure 4.0 is the theoretical density for the sample; figure 5.0 is the histogram for a unimodal distribution with two stochastic components; figure 6.0 is the theoretical density for the sample; figure 7.0 is the histogram for a multimodal distribution; and figure 8.0 is the theoretical density for the sample.

```
Independent  !              Normalized Dependent Variable
Variable     !0....1....2....3....4....5....6....7....8....9....0
             !
      0.     !*****
      0.05263!********************
      0.10526!******************************
      0.15789!***************************************
      0.21053!***********************************************
      0.26316!*********************************************
      0.31579!**************************************************
      0.36842!***********************************************
      0.42105!*************************************************
      0.47368!*****************************************
      0.52632!*************************
      0.57895!**************************************************
      0.63158!*****************************
      0.68421!*********
      0.73684!*****************
      0.78947!***********
      0.84211!***
      0.89474!*****
      0.94737!****
      1.00000!
             !
```

FIGURE 3.0
UNIMODAL HISTOGRAM
ONE STOCHASTIC COMPONENT

```
Independent  !              Normalized Dependent Variable
Variable     !0....1....2....3....4....5....6....7....8....9....0
             !
      0.02000!******
      0.04000!************
      0.06000!******************
      0.08000!***********************
      0.10000!***************************
      0.12000!*******************************
      0.14000!***********************************
      0.16000!**************************************
      0.18000!*****************************************
      0.20000!*******************************************
      0.22000!*********************************************
      0.24000!***********************************************
      0.26000!************************************************
      0.28000!*************************************************
      0.30000!**************************************************
      0.32000!**************************************************
      0.34000!**************************************************
      0.36000!**************************************************
      0.38000!*************************************************
      0.40000!*************************************************
      0.42000!************************************************
      0.44000!***********************************************
      0.46000!*********************************************
      0.48000!********************************************
      0.50000!******************************************
      0.52000!****************************************
      0.54000!***************************************
      0.56000!*************************************
      0.58000!***********************************
      0.60000!********************************
      0.62000!******************************
      0.64000!****************************
      0.66000!**************************
      0.68000!************************
      0.70000!*********************
      0.72000!*******************
      0.74000!*****************
      0.76000!***************
      0.78000!*************
      0.80000!***********
      0.82000!*********
      0.84000!*******
      0.86000!******
      0.88000!****
      0.90000!***
      0.92000!**
      0.94000!*
      0.96000!*
      0.98000!
             !
```

FIGURE 4.0
THEORETICAL PROBABILITY DENSITY
UNIMODAL WITH ONE STOCHASTIC COMPONENT

```
Independent   !              Normalized Dependent Variable
Variable      !0....1....2....3....4....5....6....7....8....9....0
              !
       0.     !
       0.05263!*- ,***
       0.10526!*********
       0.15789!****************
       0.21053!*************************
       0.26316!*******************************
       0.31579!**********************
       0.36842!**********************
       0.42105!**************************************************
       0.47368!*************************
       0.52632!******************************************
       0.57895!***************
       0.63158!***********************************
       0.68421!********************
       0.73684!*************
       0.78947!**********
       0.84211!**********
       0.89474!********
       0.94737!*****
       1.00000!
              !
```

FIGURE 5.0
UNIMODAL HISTOGRAM
TWO STOCHASTIC COMPONENTS

```
 Independent  !            Normalized Dependent Variable
Variable      !0....1....2....3....4....5....6....7....8....9....0
              !
      0.02000!*
      0.04000!**
      0.06000!****
      0.08000!*******
      0.10000!**********
      0.12000!**************
      0.14000!******************
      0.16000!**********************
      0.18000!*************************
      0.20000!*****************************
      0.22000!********************************
      0.24000!***********************************
      0.26000!**************************************
      0.28000!****************************************
      0.30000!******************************************
      0.32000!*******************************************
      0.34000!*********************************************
      0.36000!**********************************************
      0.38000!************************************************
      0.40000!*************************************************
      0.42000!**************************************************
      0.44000!**************************************************
      0.46000!**************************************************
      0.48000!*************************************************
      0.50000!************************************************
      0.52000!***********************************************
      0.54000!*********************************************
      0.56000!*******************************************
      0.58000!*****************************************
      0.60000!**************************************
      0.62000!************************************
      0.64000!*********************************
      0.66000!*******************************
      0.68000!*****************************
      0.70000!****************************
      0.72000!***************************
      0.74000!*************************
      0.76000!************************
      0.78000!**********************
      0.80000!*******************
      0.82000!*****************
      0.84000!**************
      0.86000!************
      0.88000!*********
      0.90000!*******
      0.92000!*****
      0.94000!***
      0.96000!*
      0.98000!
             !
```

FIGURE 6.0
THEORETICAL DISTRIBUTION
UNIMODAL DENSITY WITH
TWO STOCHASTIC COMPONENTS

```
Independent   |              Normalized Dependent Variable
Variable      |0....1....2....3....4....5....6....7....8....9....0
              |
      0.      |
      0.05263|***
      0.10526|*********
      0.15789|***************
      0.21053|**********************************
      0.26316|****************************************
      0.31579|**************************************************
      0.36842|********************************************
      0.42105|************************************************
      0.47368!************************************************
      0.52632!*****************************************
      0.57895|*******************************************
      0.63158|******************************
      0.68421|*****************************
      0.73684|*****************************
      0.84211|*************************
      0.78947|********
      0.89474|******
      0.94737|*
      1.00000|
             |
```

FIGURE 7.0
MULTIMODAL HISTOGRAM

```
Independent  !               Normalized Dependent Variable
Variable     !0....1....2....3....4....5....6....7....8....9....0
             !
      0.02000!
      0.04000!
      0.06000!
      0.08000!*
      0.10000!***
      0.12000!*****
      0.14000!********
      0.16000!************
      0.18000!*****************
      0.20000!**********************
      0.22000!****************************
      0.24000!*********************************
      0.26000!**************************************
      0.28000!******************************************
      0.30000!**********************************************
      0.32000!************************************************
      0.34000!**************************************************
      0.36000!**************************************************
      0.38000!*************************************************
      0.40000!************************************************
      0.42000!**********************************************
      0.44000!*******************************************
      0.46000!*****************************************
      0.48000!***************************************
      0.50000!*************************************
      0.52000!***********************************
      0.54000!**********************************
      0.56000!**********************************
      0.58000!*********************************
      0.60000!*********************************
      0.62000!**********************************
      0.64000!**********************************
      0.66000!*********************************
      0.68000!********************************
      0.70000!*******************************
      0.72000!****************************
      0.74000!*************************
      0.76000!**********************
      0.78000!******************
      0.80000!***************
      0.82000!***********
      0.84000!********
      0.86000!******
      0.88000!***
      0.90000!**
      0.92000!*
      0.94000!
      0.96000!
      0.98000!
             !
```

FIGURE 8.0
THEORETICAL DENSITY
MULTIMODAL

4. Availability of Neural Networks:

There are three ways of obtaining working versions of the statistical aids neural networks:

a. NETS version 2 is capable of generating delivery versions that can be installed on PCs.

b. The networks can be implemented in standard computer programs written in high-level languages.

c. The neural networks have been incorporated into a 'statistical expert system' (DESCR.STATS) that is available on the PRIME computer.

The incorporation of the neural networks into the 'statistical expert system,' is explained in appendix A and a listing of the computer program is shown in appendix B.

5. Results:

a. Modality:

The training set for modality, that furnished the most accurate test results, consisted of: twelve unimodal examples with one stochastic component, nine unimodal examples with two stochastic components, and ten multimodal examples. This network, when challenged with unknown beta distribution test cases, correctly identified 39 out of 40 cases, an accuracy of 97.5 percent. This same network, when challenged with test cases generated using other distributions, had an accuracy of 91 percent. These accuracies exceed the accuracy of the current statistical method for determining modality of sample data distributions. The accuracy of the network, trained with idealized sample data which was tested using noisy beta distribution data, was identical to that trained on the noisy data.

b. Number of Stochastic Components:

The training set of stochastic components, that furnished the most accurate test results consisted of: thirteen unimodal examples with one stochastic component and nine unimodal examples with two components. This network, when challenged with unknown beta distribution test cases, correctly identified 36 out of 40 cases, an accuracy of 90 percent. Two of the errors were the designation of one stochastic component as two, and the other two errors were the designation of two stochastic components as one. The same network, when challenged with test cases generated using other distributions, had an accuracy of 81 percent. The network trained on idealized data, when tested with noisy beta distribution data, was less accurate than the one trained on noisy data.

c. Combined Modality and Components:

A network was trained to furnish the combined answers, modality and number of components when presented with a set of unknown data. The best network, trained on noisy data, furnished correct answers for modality in 37 out of 40 tests; for number of components it furnished correct answers in 35 out of 40 tests. The network did not furnish any combined errors, a wrong answer for modality combined with a wrong answer for number of components. The network trained with idealized data gave the same number of incorrect answers for modality, 3 out of 40, but it also furnished 14 incorrect answers out of 40 tests for number of components. This network did not furnish any combined errors.

d. Number of Data Points.

The number of data points in the test sample distributions was varied between 150 and 3200. Within this range the accuracy of the neural networks did not appear to depend on sample size.

e. Availability of Neural Networks:

The neural networks developed for modality and components are available on the PRIME computer. It is also possible to develop delivery copies of the networks for use on PCs.

6. Conclusions:

a. The neural networks developed using NETS are capable of accurately estimating modality and stochastic components from sample data.

b. The number of data points in a sample can be as small as 150.

c. Neural networks that furnish just one statistical attribute were more accurate than the one that furnished more than one.

d. Currently the modality network and the component network are available on the PRIME computer.

APPENDIX A

Implementation of Neural Networks Into An Expert System

1. Implementation of Neural Networks into an ES

A major effort was required to implement neural networks for identifying univariate data features into the statistical expert system DESCR.STATS. A stand-alone program (RUN.1D.NET) was written to verify and validate the networks for identifying modality and number of stochastic components. Mr. Schlenker discovered that the error rates of both networks are quite sensitive to the histogram limits for the data. Of course, this is not a problem for distributions, such as beta, whose domain is well defined. However, for distributions with infinite or semi-infinite (bounded on one side) domains, assigning proper limits for the neural network is a nontrivial problem. After a bit of computational experience, he found some heuristics for calculating upper and lower limits which work well with both unimodal, one-component data and bimodal data. Of interest is the fact that the error rates for types of distributions such as normal, logistic, Weibull, and lognormal are not much greater than for the beta (and mixtures of betas), which was used exclusively in training the networks. The algorithm for assigning histogram limits makes use of the following sample statistics: average, standard deviation, minimum, maximum, and the first and third quartiles.

2. Algorithm for Histogram Limits for Neural Networks

The method for calculating the lower (XHMIN) and upper (XHMAX) limits of the histogram is sketched as follows:

If the random variable (X) represents a proportion or probability,

XHMIN = 0 and XHMAX = 1.

Otherwise, if X is strictly positive (but not a proportion), the lower limit is obtained by

XHMIN=max(0, AVGX - 3 STDV), if AVGX - 4.5 SDV < 0,

where AVGX and STDV are the sample average and standard deviation. When this condition does not hold, XHMIN is calculated as shown below. The upper histogram limit for this case is calculated in the same manner as that shown below for the case in which X may not be positive. If X is effectively unbounded or, possibly, capable of taking negative values, the lower and upper limits are calculated as follows.

XHMIN = max(XHMIN0, XHMIN1, XHMIN2)

and

XHMAX = min(XHMAX0, XHMAX1, XHMAX2),

where the alternatives XHMIN0, XHMIN1, etc. are given in terms of the minimum (XMIN) and maximum (XMAX) of the sample as well as the sample average (AVGX), standard deviation (STDV), lower quartile (X.25), and upper quartile (X.75).

$XHMIN0 = AVGX - 4.5\ STDV$, if this value < XMIN.

Otherwise,

$XHMIN0 = XHMIN0 - 1.1(XHMIN0 - XMIN)$.

Always

$XHMIN1 = XMIN - 0.5\ STDV$

$XHMIN2 = XMIN - 0.2(X.25 - XMIN)$.

The alternative upper limits are

$XHMAX0 = AVGX + 4.5\ STDV$, if this value > XMAX.

Otherwise,

$XHMAX0 = XHMAX0 + 1.1(XMAX - XHMAX0)$.

Always

$XHMAX1 = XMAX + 0.5\ STDV$

$XHMAX2 = XMAX + 0.2(XMAX - X.75)$.

This algorithm produces histogram limits which conservatively bound the values XMIN and XMAX.

3. Testing of the Neural Networks

Two networks with the same architecture, but different weights, are used to obtain (a) an indication of multimodality and (b) an indication that the random variable comes from a mixture model. These two attributes of each random variable tested are displayed in tables A-1 and A-2. If the output node for attribute (a) is < or equal to (le) 0.5, the data are declared unimodal; otherwise, they are multimodal. If the output node for attribute (b) is less than 0.5, only one stochastic component is identified. In these tables, respective identification of attributes is designated c, for correct, and *, for incorrect. Histograms do not always display the same attributes as the population density. For example, a random variable with two modes in the density may have a clearly unimodal histogram. In all cases, errors in modality and stochastic components are declared if the population density is not correctly identified. Three sample sizes--800, 400, 200--are used for each random number seed and for each set of population parameters. In most cases, if the results for a sample of 800 are correct, the results for the smaller samples are as well. Mixtures of normal and logistic random variables are found to be classified nearly as well as beta mixtures on which the networks were trained.

TABLE A-1. Results of Numerical Experiments on Testing Neural Networks With Unimodal, One-Component Distributions

Random Number Seed Indices 1 and 5

Run	Result	Dist'n	Mean	Std Dev	Sample	Seed
1	c c	lognorm	0.47	0.187	800	5
2	c c				400	
3	c c				200	
4	c *	weibull			800	
5	c c				400	
6	c c				200	
7	c *	gamma			800	
8	c c				400	
9	c *				200	
10	c c	normal			800	
11	c c				400	
12	c c				200	
13	c c	logistic			800	
14	c c				400	
15	c c				200	
16	c c	lognorm	0.47	0.170	800	
17	c c				400	
18	c c				200	
19	c c	lognorm	0.20	0.170	800	
20	c c				400	
21	c c				200	
22	c c	lognorm	0.47	0.270	800	
23	c c				400	
24	c c				200	
25	c c	weibull			800	
26	c c				400	
27	c c				200	
28	c c	beta	0.20	0.170	800	
29	c c				400	
30	c c				200	
31	c c	beta	0.47	0.270	800	
32	c c				400	
33	c c				200	
34	c *	beta	0.47	0.170	800	
35	c *				400	
36	* *				200	
37	c c	beta	0.67	0.270	800	
38	c c				400	
39	c c				200	
40	c *	weibull			800	
41	c *				400	
42	c c				200	
43	c c	logistic			800	
44	c c				400	
45	c c				200	

Continued on next page.

TABLE A-1. Results of Numerical Experiments in Testing Neural Networks With Unimodal, One-Component Distributions (Continuation)

Run	Result	Dist'n	Mean	Std Dev	Sample	Seed
46	c *	beta	0.47	0.170	800	1
47	c *				400	
48	c *				200	
49	c *	weibull			800	
50	c *				400	
51	c *				200	
52	c c	beta	0.47	0.270	800	
53	c c				400	
54	c c				200	
55	c c	beta	0.75	0.170	800	
56	c c				400	
57	c c				200	

The error rates for the two attributes are: 1/57 (< 2 percent), for modes, and 13/57 (23 percent) for stochastic components. The error rate for the latter attribute is seen to be larger than that estimated in the preliminary tests in which beta mixtures were used exclusively.

TABLE A-2. Results of Numerical Experiments in Testing Neural Networks With Unimodal and Bimodal, Two-Component Distributions

					Component 1		Component 2			
Run	Modes	Result	Dist'n	Mix	Mean	S D	Mean	S D	Sample	Seed
1	2	c c	normal	0.5	0.35	0.10	0.65	0.10	800	5
2		c c							400	
3		c c							200	
4	2	c c		0.7	0.35	0.10	0.65	0.10	800	
5		c c							400	
6		c c							200	
7	2	c c	logistic	0.7	0.35	0.10	0.65	0.10	800	
8		c c							400	
9		c c							200	
10	2 [a]	* c		0.5	0.35	0.15	0.65	0.15	800	
11		* c							400	
12		* c							200	
13	1	c *		0.6	0.35	0.20	0.65	0.20	800	
14		c *							400	
15		c *							200	
16	2	c c	beta[b]	0.5	0.35	0.09	0.65	0.09	800	
17		c c							400	
18		c c							200	
19	1	c c	logistic	0.6	0.35	0.20	0.65	0.20	400	1
20		c c							200	
21	2	c c	beta	0.5	0.35	0.097	0.65	0.097	800	
22		c c							400	
23		c c							200	
24	2	c c		0.5	0.35	0.106	0.65	0.106	800	
25		c c							400	
26		c c							200	
27	2 [a]	c c		0.6	0.35	0.106	0.65	0.106	800	
28		* c							400	
29		c c							200	
30	2 [a]	c c		0.4	0.35	0.106	0.65	0.106	800	
31		c c							400	
32		c c							200	
33	1	c c		0.5	0.35	0.117	0.65	0.117	800	
34		c c							400	
35		c c							200	
36	1	c c		0.5	0.35	0.132	0.65	0.132	800	
37		c c							400	
38		c c							200	

Continued on next page.

[a] Histogram is distinctly unimodal, altho density is bimodal.
[b] Threshold parameter of first beta component is 0 with upper limit of 0.7. Threshold of second beta component is 0.3 with upper limit of 1.0.

TABLE A-2. Results of Numerical Experiments in Testing Neural Networks With Unimodal and Bimodal, Two-Component Distributions (Continuation)

					Component 1		Component 2			
Run	Modes	Result	Dist'n	Mix	Mean	S D	Mean	S D	Sample	Seed
39	1	c c		0.5	0.35	0.156	0.65	0.156	800	
40		c c							400	
41	1	c c		0.6	0.35	0.117	0.65	0.117	800	
42		c c							400	
43		c c							200	
44	1	c c		0.7	0.35	0.117	0.65	0.117	800	
45		c c							400	
46		c c							200	
47	2	c c	logistic	0.7	0.35	0.117	0.65	0.117	800	
48		c c							400	
49		c c							200	
50	2 [a]	* *		0.5	0.35	0.156	0.65	0.156	800	
51		* c							400	
52		* *							200	
53	1	c *		0.5	0.35	0.200	0.65	0.200	800	
54		c *							400	
55		c *							200	

[a] Two modes are barely evident in the density, but are not clearly evident in a typical histogram.

Neural-net error rates for the two-component distributions tested are 7/55 (13 percent), for identifying modes, and 8/55 (14 percent) for identifying stochastic components.

4. Performance Summary and Conclusions

In total 112 data sets were used to challenge the identification accuracy of these two neural networks. The net which identifies modality of the population density was correct in 93 percent (104/112) of the sets. The net for identifying stochastic components did not perform quite as well, yielding 81 percent (91/112) correct responses. Of course, these results depend strongly upon the choice of distributions tested and to a lesser extent upon the sample of data drawn from each. The performance of the net for identifying modality was shown to be better than that for an alternative, statistically based method. There is no known alternative for the network which identifies stochastic components.

APPENDIX B

Neural Networks for a
Statistical Advisor System

Simscript Computer Source Program

Neural Networks for a Statistical Advisor System

Simscript Computer Source Program

```
PREAMBLE ''RUN.1D.NET
NORMALLY MODE IS REAL
DEFINE SIGMOID AS A REAL FUNCTION GIVEN 2 ARGUMENTS
END ''PREAMBLE


     MAIN ''RUN.1D.NET
''
''   DRIVER FOR THE ROUTINE N1D.NET FOR ANALYSIS OF UNIVARIATE DATA.
''   Program has been augmented to read a data file of random variables,
''   create a histogram and normalize it for the neural net, and call the
''   program N1D.NET with appropriate input vector.
''
''Required Programs/Functions:  DYQUANT, UPDT.HIST, SIGMOID.
''
     DEFINE ANSWER,FILINAM,TITLE AS TEXT VARIABLES
     DEFINE FLAGN,FLAGP,I,ITRUNC,J,N,NCELLS,NCOUNT,NDATA AS INTEGER
     VARIABLES
     DEFINE HISTV,NXV AS INTEGER, 1-DIMENSIONAL ARRAYS
     RESERVE NXV(*) AS 5
     DEFINE NODEV,QV,XINV AS REAL, 1-DIMENSIONAL ARRAYS
     RESERVE QV(*) AS 5 ''MARKERS FOR QUANTILES OF XINV
     LET NCELLS=20
     LET LSDL=3.0
     LET USDL=4.5
     LET WLIM=1.1
     RESERVE HISTV(*),NODEV(*) AS NCELLS
     LET LINES.V=9999
     LET FILINAM = "RV.DATA"
'LO'PRINT 1 LINE WITH FILINAM
     THUS
INPUT THE NUMBER OF DATA POINTS TO BE READ FROM ********
     READ NDATA
     RESERVE XINV(*) AS NDATA
''
''OPEN UNIT 4 FOR INPUT OF THE RANDOM-VARIABLE DATA.
''
     OPEN UNIT 4 FOR INPUT,
     OLD,
     FILE NAME IS FILINAM
     RECORD SIZE IS 80
     USE UNIT 4 FOR INPUT
     READ TITLE USING UNIT 4
     FOR I=1 TO NDATA, READ XINV(I) USING UNIT 4
     CLOSE UNIT 4 ''FOR THIS INPUT
''
''GET AVGX, STDV, XMIN, AND XMAX.
''
```

```
    LET XMIN=RINF.C
    LET XMAX= -RINF.C
    LET AVGX=0.0
    LET STDV=0.0
    FOR I=1 TO NDATA DO
        LET X=XINV(I)
        CALL DYQUANT(0.5,QV(*),I,X,NXV(*))
        LET XMIN=MIN.F(XMIN,X)
        LET XMAX=MAX.F(XMAX,X)
        ADD X TO AVGX
        ADD X**2 TO STDV
    LOOP ''OVER (I) POINTS
    LET AVGX=AVGX/NDATA
    LET STDV=(STDV - NDATA*AVGX**2)/(NDATA - 1)
    LET STDV=SQRT.F(STDV)
    IF QV(1) NE XMIN OR QV(5) NE XMAX
        PRINT 2 LINES WITH QV(1),XMIN,QV(5),XMAX
        THUS
ERROR.  INCONSISTENCY IN CALCULATING MIN AND MAX.  First of pair obtained
in routine DYQUANT: ( ***.****,  ***.****) and ( ***.****,  ***.****).
        STOP
    OTHERWISE
    SKIP 1 LINE
    PRINT 4 LINES WITH XMIN,XMAX,AVGX,STDV,QV(2),QV(3),QV(4)
    THUS
Min X  =  ****.****    Max X =  ****.****
Avg X  =  ****.****    S D X =  ****.****
X.25   =  ****.****    X.50  =  ****.****   X.75 = ****.****

    IF XMIN LE 0.0
        LET FLAGN=1 ''INDICATING THAT RV CAN BE NEGATIVE
        LET FLAGP=0 ''INDICATING THAT RV IS NOT A PROPORTION
        GO TO J0
    OTHERWISE
    PRINT 2 LINES THUS
CAN THE RANDOM VARIABLE TAKE ON NEGATIVE VALUES OR BE TREATED AS
EFFECTIVELY UNBOUNDED FROM BELOW?  (YES OR NO).
    READ ANSWER
    IF SUBSTR.F(ANSWER,1,1) = 'Y' OR SUBSTR.F(ANSWER,1,1) = 'y'
        LET FLAGN=1
        LET FLAGP=0
        GO TO J0
    OTHERWISE
    LET FLAGN=0
    IF XMAX < 1.0
        PRINT 2 LINES THUS
DOES THE RANDOM VARIABLE REPRESENT A PROPORTION (PROBABILITY)?  (Y OR N).
That is, must the random variable lie within the range (0, 1)?
        READ ANSWER
        IF SUBSTR.F(ANSWER,1,1) = 'Y' OR SUBSTR.F(ANSWER,1,1) = 'y'
            LET FLAGP=1
            GO TO J0
        OTHERWISE
    ALWAYS
```

```
    LET FLAGP=0
''
''DETERMINE HISTOGRAM LIMITS.
''
'J0'IF FLAGP=1
        LET XHMIN=0.0
        LET XHMAX=1.0
        GO TO J1
    OTHERWISE
    IF FLAGN=0 ''X IS STRICTLY POSITIVE
        IF AVGX - USDL*STDV > 0.0
''
''TREAT X AS EFFECTIVELY UNBOUNDED IN CALCULATING HISTO LIMITS.
''
            GO TO J3
        OTHERWISE
        LET XHMIN0=AVGX - LSDL*STDV
        LET XHMIN1=XMIN - 0.5*STDV
        LET XHMIN2=XMIN - 0.2*(QV(2) - XMIN)
        LET XHMIN=MAX.F(0.0,XHMIN0,XHMIN1,XHMIN2)
        LET XHMAX=AVGX + USDL*STDV
        IF XHMAX < XMAX
            LET XHMAX=XHMAX + WLIM*(XMAX-XHMAX)
        ALWAYS
''
''CONSTRAINTS ON THE LIMITS.
''
        LET XHMAX1=XMAX + 0.5*STDV
        LET XHMAX2=XMAX + 0.2*(XMAX - QV(4))
        LET XHMAX=MIN.F(XHMAX,XHMAX1,XHMAX2)
        GO TO J1
    OTHERWISE ''RANGE OF X INCLUDES NEGATIVES OR IS EFFECTIVELY UNBOUNDED
'J3'LET XHMIN=AVGX - USDL*STDV
    IF XMIN < XHMIN
        LET XHMIN=XHMIN - WLIM*(XHMIN-XMIN)
    ALWAYS
    LET XHMAX=AVGX + USDL*STDV
    IF XMAX > XHMAX
        LET XHMAX=XHMAX + WLIM*(XMAX-XHMAX)
    ALWAYS
''
''CONSTRAINTS ON THE LIMITS.
''
    LET XHMIN1=XMIN - 0.5*STDV
    LET XHMIN2=XMIN - 0.2*(QV(2) - XMIN)
    LET XHMIN=MAX.F(XHMIN,XHMIN1,XHMIN2)
    LET XHMAX1=XMAX + 0.5*STDV
    LET XHMAX2=XMAX + 0.2*(XMAX - QV(4))
    LET XHMAX=MIN.F(XHMAX,XHMAX1,XHMAX2)
'J1'LET DELX=(XHMAX - XHMIN)/NCELLS
    IF DELX LE 0.0
        PRINT 1 LINE THUS
TROUBLE WITH INPUT DATA.  Calculated DELX in histogram is 0.
        STOP
```

```
       OTHERWISE
       LET ITRUNC=0
       LET NCOUNT=0
       FOR I=1 TO NDATA DO
           LET X=XINV(I)
   ''
   ''LOCATE VALUES OF X WITH RESP TO HISTO CELLS AND INCREMENT CELL FREQS.
   ''
           CALL UPDT.HIST(X,NCELLS,ITRUNC,XHMIN,DELX,HISTV(*)) YIELDING N
           ADD N TO NCOUNT
       LOOP ''OVER (I) POINTS
       IF FLAGP=1
           GO TO J4
       OTHERWISE
   ''
   ''CHECK FOR EXCEPTIONAL COUNTS IN LAST CELL.
   ''
       IF 2*HISTV(NCELLS) > HISTV(NCELLS-1) + HISTV(NCELLS-2) AND FLAGP NE 1
   ''
   ''UPPER-TAIL OUTLIERS MAY BE PRESENT.  TRUNCATE VALUES ABOVE XHMAX.
   ''
           LET ITRUNC=1
           LET NCOUNT=0
           FOR I=1 TO NCELLS, LET HISTV(I)=0
           FOR I=1 TO NDATA DO
               LET X=XINV(I)
               CALL UPDT.HIST(X,NCELLS,ITRUNC,XHMIN,DELX,HISTV(*)) YIELDING N
               ADD N TO NCOUNT
           LOOP ''OVER (I) DATA POINTS
   ''
   ''TRANSFORM HISTO FREQUENCIES TO GET NODAL VALUES FOR A NEURAL NET.
   ''
       ALWAYS
   'J4'IF NCOUNT LE 0
           PRINT 1 LINE THUS
TROUBLE IN GETTING HISTOGRAM COUNT.  TOTAL COUNT = 0.
           STOP
       OTHERWISE
       FOR I=1 TO NCELLS DO
           LET NODEV(I)=HISTV(I)/NCOUNT + 0.1
           IF NODEV(I) > 0.9
               PRINT 2 LINES WITH I,NODEV(I)
               THUS
TROUBLE.  Error in normalizing for neural-net input nodes.
NODEV(**) =  *.****
               STOP
           OTHERWISE
       LOOP ''OVER (I) HISTOGRAM CELLS
   ''
   ''DETERMINE WHETHER DENSITY OF X IS UNIMODAL OR MULTIMODAL.
   ''
       PRINT 3 LINES WITH TITLE,XHMIN,XHMAX,NCOUNT,NDATA
       THUS
```

```
 SUBJECT: ******************************************************
 Min X in Histogram:  **.****   Max X in Histogram:  **.****
 Total Frequency in Histogram:  ****  Total RVs Generated:  ****
        CALL N1D.NET GIVEN 1,NODEV(*) YIELDING UMOUT
        PRINT 1 LINE WITH UMOUT
        THUS
 Output node for modality of density is  **.****
    ''
    ''DETERMINE WHETHER DENSITY REPRESENTS A MIXTURE MODEL.
    ''
        SKIP 1 LINE
        CALL N1D.NET GIVEN 2,NODEV(*) YIELDING UMOUT
        PRINT 1 LINE WITH UMOUT
        THUS
 Output node for stochastic components is *.****
        SKIP 1 LINE
    ''  PRINT 1 LINE THUS
    ''  OTHER INPUTS WANTED?  (Y OR N).
    ''  READ ANSWER
    ''  IF SUBSTR.F(ANSWER,1,1) = 'Y' OR SUBSTR.F(ANSWER,1,1) = 'y'
    ''      RELEASE XINV(*)
    ''      GO TO LO
    ''  OTHERWISE
        STOP
    END ''RUN.1D.NET

        ROUTINE N1D.NET GIVEN IA, XINV YIELDING UMOUT
    ''
    ''Program calculates attributes of a one-dimensional density function from
    ''N1-cell histogram (scaled), using a neural network.  Input layer of the
    ''network contains N1 nodes; the middle layer contains N2 nodes; and the
    ''output layer contains one node.  Nodal architecture permits each input
    ''node to be connected to each node of the middle layer.  The output node
    ''is, in turn, connected to each node of the middle layer.  The attribute
    ''represented by the output nodal value UMOUT depends upon the value of
    ''the integer flag IA.  If IA = 1, the output encodes modality of the
    ''probability density--0.1, for unimodal and 0.9 otherwise.  If IA = 2,
    ''the output node encodes number of stochastic components in density
    ''mixture model--0.1 for one component and 0.9 otherwise.  Input nodal
    ''values are contained in vector XINV, representing scaled frequencies
    ''in a histogram of the data.  Use of program for data samples less than
    ''about 150 is not recommended.  The two-argument SIGMOID function is
    ''required.  Also, two files of weights are necessary--NETS.WT2.DATA
    ''and NETS.WT4.DATA.
    ''
        DEFINE ANSWER,FILWNAM AS TEXT VARIABLES
        DEFINE I,IA,IPRINT,J,K,L,LAYERS,M,N,NWTS AS INTEGER VARIABLES
        DEFINE NV AS AN INTEGER, 1-DIMENSIONAL ARRAY
        DEFINE PARMV,XINV AS REAL, 1-DIMENSIONAL ARRAYS
        DEFINE XA AS A REAL, 2-DIMENSIONAL ARRAY
        DEFINE WTA AS A REAL, 3-DIMENSIONAL ARRAY
    ''
    ''DEFINITIONS OF ARRAYS:
    ''
```

```
''XINV(I) ___  VALUE OF THE Ith NODE IN THE INPUT LAYER (NUMBER 1).
''XA(K,I) ___  VALUE OF THE Ith NODE IN LAYER K, K=1 (input) TO LAYERS.
''WTA(K,I,J) _ WEIGHT ON ARC BETW Ith NODE OF LAYER K+1 AND Jth NODE OF
''             LAYER K, FOR K=1 TO LAYERS-1, I=1 TO NV(K+1),J=1 TO NV(K).
''
''
''DEFAULT VALUES.
''
''
''THE FOLLOWING FILES CONTAIN WEIGHTS FOR 1-D NETWORKS OF 3 LAYERS EACH
''WITH LAYER 1 HAVING 20 INPUT NODES, LAYER 2 (MIDDLE LAYER)
''CONTAINING 7 NODES, AND LAYER 3 CONTAINING A SINGLE OUTPUT NODE.
''NETS.WT2.DATA _ to determine number of stochastic components.  This
''                weight set has proven successful in all instances of
''                unimodal data with one component.  In those instances of
''                challenge sets of bimodal data with two components, the
''                weights yield good predictability.  In the case of uni-
''                modal densities with two components, the weights have
''                been less successful than in the other cases.
''NETS.WT4.DATA _ to determine modality of the population density.  If
''                value of the output node exceeds 0.5, a multimodal
''                density is implied; otherwise, density is predicted as
                  unimodal.
''
    LET IPRINT=0 ''FOR NO PRINT
    LET LAYERS=3
    RESERVE NV(*) AS LAYERS
    RESERVE PARMV(*) AS 4 ''PARAMETERS OF THE SIGMOID FUNCTION
    LET PARMV(1)=0.0 ''LOCATION
    LET PARMV(2)=1.0 ''SCALE
    LET PARMV(3)=0.0 ''MIN VALUE
    LET PARMV(4)=1.0 ''MAX VALUE
    RESERVE XA(*,*) AS LAYERS BY *
    RESERVE WTA(*,*,*) AS LAYERS-1  BY *
    LET NV(1)=20  ''DEFAULT
    LET NV(2)=7   ''DEFAULT
    LET NV(3)=1   ''DEFAULT
''
''CHECK FOR VALID INPUTS.
''
    IF IA LE 0 OR IA > 2
        PRINT 1 LINE WITH IA
        THUS
INVALID INPUT TO ROUTINE N1D.NET.  IA = ***
        STOP
    OTHERWISE
    IF DIM.F(XINV(*)) NE 20
        PRINT 1 LINE THUS
INVALID DIMENSION OF THE INPUT VECTOR IN ROUTINE N1D.NET.
        STOP
    OTHERWISE
    IF IA=1
        LET FILWNAM = 'NETS.WT4.DATA'
    OTHERWISE
```

```
        LET FILWNAM = 'NETS.WT2.DATA'
    ALWAYS
''
''RESERVE ARRAYS.
''
    FOR K=1 TO LAYERS, RESERVE XA(K,*) AS NV(K)
    FOR K=1 TO LAYERS-1, RESERVE WTA(K,*,*) AS NV(K+1) BY NV(K)
''
''MOVE INPUT NODAL VALUES TO XA.
''
    FOR I=1 TO NV(1), LET XA(1,I)=XINV(I)
''
''OPEN UNIT 4 FOR NETWORK WEIGHTS.
''
    OPEN UNIT 4 FOR INPUT,
    OLD,
    FILE NAME IS FILWNAM
    RECORD SIZE IS 80
    USE UNIT 4 FOR INPUT
    LET NWTS=0
    FOR K=1 TO LAYERS-1 DO
        LET M=NV(K+1)
        LET N=NV(K)
        ADD M*N TO NWTS
        FOR I=1 TO M, FOR J=1 TO N, READ WTA(K,I,J) USING UNIT 4
    LOOP ''OVER (K) LAYERS
    CLOSE UNIT 4
    IF IPRINT=1
        SKIP 2 LINES
        PRINT 5 LINES WITH FILWNAM,LAYERS,NWTS
        THUS
 File name for network weights:    ****************************************

 NETWORK ARCHITECTURE:
 Total Layers in Network:  **
 Total Number of Weights:  ******
    ALWAYS
    FOR K=1 TO LAYERS-1 DO
        LET M=NV(K+1)
        LET N=NV(K)
        FOR I=1 TO M DO
            LET SUM=0.0
            FOR J=1 TO N DO
                ADD WTA(K,I,J)*XA(K,J) TO SUM
            LOOP ''OVER (J) BOTTOM NODES
            LET XA(K+1,I)=SIGMOID(SUM,PARMV(*)) ''K+1ST LAYER NODES
        LOOP ''OVER TOP NODES
    LOOP ''OVER (K) LAYERS
    LET UMOUT=XA(LAYERS,1)
''
''PRINT OUTPUT.
''
    IF IPRINT=1
        IF IA=1
```

```
              LET ANSWER = 'modality of density'
          OTHERWISE
              LET ANSWER = 'stochastic components' .
          ALWAYS
          SKIP 1 LINE
          PRINT 1 LINE WITH ANSWER,UMOUT
          THUS
 Output node for *********************  =  **.****
      ALWAYS
      RELEASE PARMV(*)
      RELEASE NV(*)
      RELEASE XA(*,*)
      RELEASE WTA(*,*,*)
      RETURN
  END ''N1D.NET
```

```
  FUNCTION SIGMOID (X, PARMV)
  ''
  ''Function calculates the sigmoidal value with argument X and parameter
  ''vector PARMV.  There are four elements in PARMV(*):  (1) the location
  ''or bias parameter for X, (2) scale parameter for X, (3) min value of
  ''the function, and (4) max value of the function.  The function, f(x),
  ''is given by:
  ''
  ''   (PARMV(4)-PARMV(3))/(1 + exp -[(X-PARMV(1))/PARMV(2)]) + PARMV(3) .
  ''
  ''The function may be used as a smooth limiter for variable X, taking on
  ''value PARMV(3) + (PARMV(4)-PARMV(3))/2  when z = (X-PARMV(1))/PARMV(2)
  ''is zero, and approaching PARMV(3) as z approaches -inf and PARMV(4)
  ''as X approaches +inf.
  ''Note that f(z) is quite linear over the region -0.5 < z < 0.5. A
  ''noticeable nonlinearity develops for abs(z) > 0.7, approximately.
  ''
      DEFINE FLAGU AS AN INTEGER VARIABLE
      DEFINE PARMV AS A REAL, 1-DIMENSIONAL ARRAY
      LET FLAGU=1
      IF FLAGU=1
          RETURN WITH 1.0/(1.0 + EXP.F(-X))
      OTHERWISE
      IF PARMV(2) LE 0.0
          PRINT 1 LINE THUS
 INPUT ERROR TO FUNCTION SIGMOID.  SCALE PARAMETER IS L.E. 0.
          STOP
      OTHERWISE
      LET Z=(X - PARMV(1))/PARMV(2)
      RETURN WITH PARMV(3) + (PARMV(4) - PARMV(3))/(1.0 + EXP.F(-Z))
      END ''FUNCTION SIGMOID
```

```
  ROUTINE UPDT.HIST(X,NCELLS,ITRUNC,XMIN,DELX,HISTV) YIELDING NCOUNT
  ''
  ''Routine updates frequency count in the histogram HISTV to include the
  ''random variable X.  Number of cells in the histogram is NCELLS.  The
  ''min (lower-limit) of the independent variable is XMIN and cell width
  ''or independent-variable increment is DELX.  Independent variable is con-
```

```
''sidered continuous and HISTV is discrete.  The vector HISTV(*) is
''dimensioned NCELLS.  A cumulative counter NCOUNT is increased by 1 if
''either of these conditions holds:  (a) flag ITRUNC is set to 0, or (b)
''X le maximum cell upper bound, XMIN + NCELLS*DELX.
''
    DEFINE ITRUNC,K,NCELLS,NCOUNT AS INTEGER VARIABLES
    DEFINE HISTV AS AN INTEGER, 1-DIMENSIONAL ARRAY
    FOR K=1 TO NCELLS DO
        LET XUP=XMIN + DELX*K
        IF X LE XUP
            ADD 1 TO HISTV(K)
            ADD 1 TO NCOUNT
            RETURN
        OTHERWISE
    LOOP ''OVER (K) HISTOGRAM CELLS
    IF ITRUNC=0
        ADD 1 TO HISTV(NCELLS)
        ADD 1 TO NCOUNT
    ALWAYS
    RETURN
END ''OF ROUTINE UPDT.HIST
```

```
ROUTINE DYQUANT (P, QV, N, XN, NXV)
''
''Program calculates the P-quantile from a set of observations presented
''dynamically (as generated).  Quantile is updated after each observation
''is made.  The number (N) of newest observation and its value (XN) are
''specified by the calling program.  Ref to method is made to an article
''in Communications of the ACM, Vol 28, No 10, Oct 1985:  Raj Jain and
''Imrich Chlamatac, 'The P-Squared Algorithm for Dynamic Calculation of
''Quantiles and Histograms Without Storing Observations', p. 1076 ff.  A
''set of five 'markers' of RV quantiles are stored in QV(*) and a set of
''five 'horizontal positions' for markers are stored in NXV(*).  Markers
''are used to locate the real-valued quantiles.  These are defined as:
''QV(1) ___ minimum value over all RV observations currently made.
''QV(2) ___ lowest middle marker, corresponding to the P/2 quantile.
''QV(3) ___ desired P quantile.
''QV(4) ___ highest middle marker, corresponding to (1 + P)/2 quantile.
''QV(5) ___ maximum value over all RV observations currently made.
''
''Routine is called after each observation is made with the number and
''value of the observation.  First five observations are sorted in ascend-
''ing order and placed in QV(*).  Next (Nth) observation is given in XN.
''A vector of 'horizontal' marker positions, NXV(*), is maintained by the
''routine.  After the Nth observation, NXV(i) = number of observations
''(approx) whose value is l.e. QV(i), i = 1, ..., 5.
''
    DEFINE I,J,K,N AS INTEGER VARIABLES
    DEFINE QV AS A REAL, 1-DIMENSIONAL ARRAY
    DEFINE DNV AS A REAL, SAVED, 1-DIMENSIONAL ARRAY
    RESERVE DNV(*) AS 5
    DEFINE NXV AS AN INTEGER, 1-DIMENSIONAL ARRAY
    IF N > 5
        GO TO LO
```

```
OTHERWISE
IF N=1
    LET QV(1)=XN
    FOR I=1 TO 5, LET NXV(I)=I
    LET DNV(1)=1.0
    RETURN
OTHERWISE
IF N=2
    IF XN > QV(1)
        LET QV(2)=XN
    OTHERWISE
        LET QV(2)=QV(1)
        LET QV(1)=XN
    ALWAYS
    RETURN
OTHERWISE
IF N=3
    IF XN < QV(1)
        LET QV(3)=QV(2)
        LET QV(2)=QV(1)
        LET QV(1)=XN
        RETURN
    OTHERWISE
    IF XN > QV(2)
        LET QV(3)=XN
        RETURN
    OTHERWISE
    LET QV(3)=QV(2)
    LET QV(2)=XN
    RETURN
OTHERWISE
IF N=4
    IF XN < QV(1)
        LET QV(4)=QV(3)
        LET QV(3)=QV(2)
        LET QV(2)=QV(1)
        LET QV(1)=XN
        RETURN
    OTHERWISE
    IF XN > QV(3)
        LET QV(4)=XN
        RETURN
    OTHERWISE
    IF XN < QV(2)
        LET QV(4)=QV(3)
        LET QV(3)=QV(2)
        LET QV(2)=XN
        RETURN
    OTHERWISE
    LET QV(4)=QV(3)
    LET QV(3)=XN
    RETURN
OTHERWISE ''N=5
IF XN < QV(1)
```

```
            FOR I=1 TO 4, LET QV(5-I+1)=QV(5-I)
            LET QV(1)=XN
            RETURN
        OTHERWISE
        IF XN > QV(4)
            LET QV(5)=XN
            RETURN
        OTHERWISE
        IF XN < QV(2)
            FOR I=1 TO 3, LET QV(5-I+1)=QV(5-I)
            LET QV(2)=XN
            RETURN
        OTHERWISE
        IF XN < QV(3)
            FOR I=1 TO 2, LET QV(5-I+1)=QV(5-I)
            LET QV(3)=XN
            RETURN
        OTHERWISE
        LET QV(5)=QV(4)
        LET QV(4)=XN
        RETURN
''
''CALCULATE THE DESIRED MARKER POSITIONS: DN2, DN3, DN4.
''
'LO'LET NM1=N-1
        LET DNV(1)=1.0
        LET DNV(2)=0.5*P*NM1+1.0
        LET DNV(3)=NM1*P+1.0
        LET DNV(4)=0.5*(1.0+P)*NM1+1.0
        LET DNV(5)=N
        IF XN < QV(1)
            LET QV(1)=XN
            LET K=1
            GO TO L1
        OTHERWISE
        IF XN > QV(5)
            LET QV(5)=XN
            LET K=4
            GO TO L1
        OTHERWISE
''
''FIND CELL K SUCH THAT QV(K) LE XN < QV(K+1)
''
        FOR K=1 TO 4 DO
            IF QV(K) LE XN AND XN < QV(K+1)
                GO TO L1
            OTHERWISE
        LOOP ''OVER K
        IF XN=QV(5)
            LET K=4
        ALWAYS
''
''INCREMENT POSITIONS OF MARKERS K+1 THRU 5.
''
```

```
'L1'FOR I=K+1 TO 5, ADD 1 TO NXV(I)
''
''ADJUST HEIGHTS OF MARKERS 2 THRU 4.
''
    FOR I=2 TO 4 DO
        LET J=0 ''TO INDICATE SIGN OF ADJUSTMENT
        LET D=DNV(I)-NXV(I)
        IF D GE 1.0 AND NXV(I+1) > NXV(I)+1
            LET J=1
            GO TO L2
        OTHERWISE
        IF D LE -1.0 AND NXV(I-1) < NXV(I)-1
            LET J= -1
'L2'        LET QI=QV(I)+J/(NXV(I+1)-NXV(I-1))*((NXV(I)-NXV(I-1)+J)*
            (QV(I+1)-QV(I))/(NXV(I+1)-NXV(I))+(NXV(I+1)-NXV(I)-J)*
            (QV(I)-QV(I-1))/(NXV(I)-NXV(I-1)))
''
''TRIAL VALUE OF QV(I) IS QI.
''
            IF QV(I-1) < QI AND QI < QV(I+1)
                LET QV(I)=QI
            OTHERWISE ''USE LINEAR FORM TO ADJUST QV(I)
                LET QV(I)=QV(I)+J*(QV(I+J)-QV(I))/(NXV(I+J)-NXV(I))
            ALWAYS
            ADD J TO NXV(I)
        ALWAYS
    LOOP ''OVER I
    RETURN
END ''ROUTINE DYQUANT
```

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